

 $Core(N, v, S) \subseteq preBS(N, v, S).$

This is true since, if $x \in Core(N, v, S)$, no agent i has any objection against any other agent j.

Let us consider the objections (P,y) of 1 against another member of $\{2,3,4,5,6,7\}$. Since the players $\{2,...,7\}$ play symmetric roles, we consider an objection (P,y) of 1 against 7 using successively as

P {1,2,3,4,5,6}, {1,2,3,4,5}, {1,2,3,4}, {1,2,3}, {1,2} and {1}. We will

look for a counter-objection of player 7 using (Q, z).

•
$$P = \{1, 2, 3, 4, 5, 6\}$$
. We need to find the payoff vector $y \in \mathbb{R}^6$ so that (P, y) is an objection. $y = \langle \alpha, \frac{1}{5} + \alpha_2, \frac{1}{5} + \alpha_3, \dots, \frac{1}{5} + \alpha_6 \rangle$.
The conditions for (P, y) to be an objection are the following:
• each agent is as well off as in $x: \alpha > -\frac{1}{5}, \alpha_i \ge 0$
• y is feasible for coalition $P: \sum_{i=2}^{6} (\alpha_i + \frac{1}{5}) + \alpha \le 1$.
w.l.o.g $0 \le \alpha_2 \le \alpha_3 \le \alpha_4 \le \alpha_5 \le \alpha_6$.
Then $\sum_{i=2}^{6} (\frac{1}{5} + \alpha_i) + \alpha = \frac{5}{5} + \sum_{i=2}^{6} \alpha_i + \alpha = 1 + \sum_{i=2}^{6} \alpha_i + \alpha \le 1$.
Then $\sum_{i=2}^{6} \alpha_i \le -\alpha < \frac{1}{5}$.
• We need to find a counter-objection for (P, y) .
claim: we can choose $Q = (2, 3, 4, 7)$ and
 $z = \langle \frac{1}{5} + \alpha_2, \frac{1}{5} + \alpha_3, \frac{1}{5} + \alpha_4, \frac{1}{5} + \alpha_5 \rangle$
 $z(Q) = \frac{1}{5} + \alpha_2 + \frac{1}{5} + \alpha_3 + \frac{1}{5} + \alpha_4 + \frac{1}{5} + \alpha_5 = \frac{4}{5} + \sum_{i=2}^{5} \alpha_i \le 1$ since
 $\sum_{i=2}^{5} \alpha_i \le \sum_{i=2}^{6} \alpha_i < \frac{1}{5}$ so z is feasible.
It is clear that $\forall i \in Q, z_i \ge x_i$ and that $\forall i \in Q \cap P, z_i \ge y_i$ \checkmark
Hence, (Q, z) is a counter-objection. \checkmark

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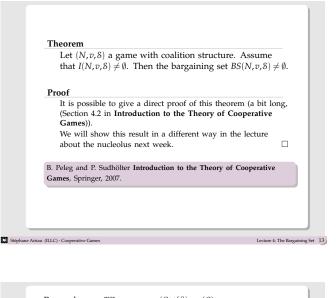
Using similar arguments, we find a counter-objection for each other objections (you might want to fill in the details at home).

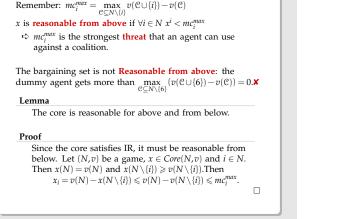
- $\begin{array}{l} \circ \ P = \{1,2,3,4\}, \ y = \langle \alpha, \frac{1}{5} + \alpha_1, \frac{1}{5} + \alpha_2, \frac{1}{5} + \alpha_3 \rangle, \ \alpha > -\frac{1}{5}, \ \alpha_i \geqslant 0, \\ \sum_{i=2}^4 \alpha_i + \alpha \leqslant \frac{2}{5} \Rightarrow \sum_{i=2}^4 \alpha_i \leqslant \frac{2}{5} \alpha < \frac{3}{5}. \end{array}$
- $Q = \{2, 5, 6, 7\}, z = \langle \frac{1}{5} + \alpha_2, \frac{1}{5}, \frac{1}{5}, \frac{1}{5} \rangle$ since $\alpha_2 \leq \frac{1}{5}$
- $|P| \leq 3 \ P = \{1, 2, 3\}, \ v(P) = 0, \ y = \langle \alpha, \alpha_1, \alpha_2 \rangle, \ \alpha > -\frac{1}{5},$ $\alpha_i \ge 0, \ \alpha_1 + \alpha_2 \le -\alpha < \frac{1}{5}$
- $Q = \{4,5,6,7\}, \, z = \langle \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5} \rangle$ will be a counter argument (1 cannot provide more than $\frac{1}{5}$ to any other agent).
- For each possible objection of 1, we found a counter-objection. Using similar arguments, we can find a counter-objection to any objection of player 7 against player 1.
- $\Rightarrow x \in preBS(N, v, S)$.

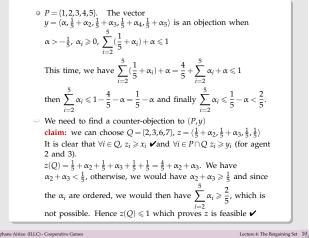
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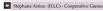
Lecture 4: The Bargaining Set 11)

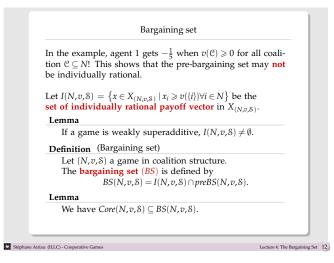
Lecture 4: The Bargaining Set 9

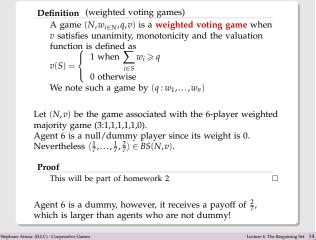




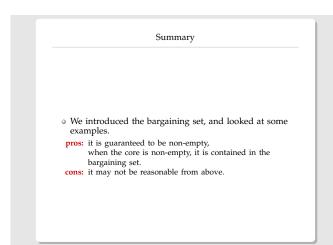








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 We will consider the Nucleolus. It can also be defined in terms of objections and counter objections, but the nature of the objection is different from the bargaining set.

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Lecture 4: The Bargaining Set 17